

# Package: mosaicCalc (via r-universe)

May 22, 2026

**Type** Package

**Title** R-Language Based Calculus Operations for Teaching

**Description** Software to support the introductory \*MOSAIC Calculus\* textbook <<https://www.mosaic-web.org/MOSAIC-Calculus/>>, one of many data- and modeling-oriented educational resources developed by Project MOSAIC (<<https://www.mosaic-web.org/>>). Provides symbolic and numerical differentiation and integration, as well as support for applied linear algebra (for data science), and differential equations/dynamics. Includes grammar-of-graphics-based functions for drawing vector fields, trajectories, etc. The software is suitable for general use, but intended mainly for teaching calculus.

**Version** 0.6.5.9004

**Depends** R (>= 3.5.0)

**Imports** calculus, Deriv, dplyr, ggformula, ggplot2, glue, grDevices, MASS, Matrix, metR (>= 0.11.0), mosaicCore (>= 0.9.2), mosaic, orthopolynom, rlang, Ryacas, sp, stats, tibble

**Suggests** testthat, knitr, palmerpenguins, rmarkdown, cubature, plotly, mosaicData

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**VignetteBuilder** knitr

**NeedsCompilation** no

**License** GPL (>=2)

**LazyLoad** true

**LazyData** true

**URL** <https://github.com/ProjectMOSAIC/mosaicCalc>

**BugReports** <https://github.com/ProjectMOSAIC/mosaicCalc/issues>

**RoxygenNote** 7.3.2

**Roxygen** list(markdown = TRUE)

**Encoding** UTF-8

**Config/pak/sysreqs** libabsl-dev libcairo2-dev cmake libfontconfig1-dev libfreetype6-dev libgdal-dev gdal-bin libgeos-dev make libicu-dev libpng-dev libuv1-dev libssl-dev libproj-dev libsqlite3-dev libudunits2-dev libx11-dev zlib1g-dev

**Repository** <https://dtkaplan.r-universe.dev>

**Date/Publication** 2026-02-21 13:44:59 UTC

**RemoteUrl** <https://github.com/ProjectMOSAIC/mosaicCalc>

**RemoteRef** HEAD

**RemoteSha** b3c30df7ced749e923f02dcf2e2d9cba03857e03

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---

Aquaman

*Box office from the movie Aquaman*

---

### Description

Earnings each weekend after the release of the movie *Aquaman*

### Usage

Aquaman

### Format

A data frame with 20 rows and variables Date, Earnings (in USD), and the count of the Weekend.

**Source**

Beth Schaubroeck at USAFA. Scraped from [https://www.boxofficemojo.com/release/r1310880001/weekend/?ref=bo\\_rl\\_tab](https://www.boxofficemojo.com/release/r1310880001/weekend/?ref=bo_rl_tab).

---

argM	<i>Find local extreme points</i>
------	----------------------------------

---

**Description**

Find local extreme points

**Usage**

```
argM(tilde, domain)
```

**Arguments**

tilde	specification of a function as in makeFun()
domain	a domain to search in.

**Details**

End-points of the domain may be included in the output.

**Value**

A data frame with values for x, the function output at those values of x, and the concavity.

**Examples**

```
argM(x^2 ~ x, domain(x=-1:1))
```

---

basis_sets	<i>Basis sets for for function approximation</i>
------------	--

---

**Description**

These functions generate the mathematical functions for three different basis sets: Fourier (sines), Legendre (orthogonal polynomials), and Splines (low-order smooth approximation)

**Usage**

```

legendre_set(df = 3, left = -1, right = 1)

legendre_M(x, df, left = -1, right = 1)

ns_set(df = 3, left = -1, right = 1)

fourier_M(x, n, fperiod = NULL)

ns_M(x, df = NULL, knots = NULL, intercept = FALSE, Boundary.knots = range(x))

fourier_set(df, left = -1, right = 1)

```

**Arguments**

df	number of basis functions to construct
left	number giving left-hand boundary of the interval
right	number giving right-hand boundary of the interval
x	inputs at which to evaluate the functions (in the <code>_M</code> functions)
n	number of fourier components to generate
fperiod	number giving the fundamental period length for the Fourier basis
knots	breakpoints that define the spline. The default is no knots; together with the natural boundary conditions this results in a basis for linear regression on x. Typical values are the mean or median for one knot, quantiles for more knots. See also <code>Boundary.knots</code> .
intercept	if TRUE, an intercept is included in the basis; default is FALSE.
<code>Boundary.knots</code>	boundary points at which to impose the natural boundary conditions and anchor the B-spline basis (default the range of the data). If both <code>knots</code> and <code>Boundary.knots</code> are supplied, the basis parameters do not depend on x. Data can extend beyond <code>Boundary.knots</code>

**Details**

For each basis, there are two different forms for the generating functions. Names ending in `_set` create a set of functions with arguments `x` and `n`, where integer `n` provides an index into the set. The same names with a `_M` suffix produce a model matrix corresponding to a specified set of `x` values. These are useful with `lm()` and similar model-building functions in the same way that `poly()` and `ns()` are useful. (`ns_M()` is just an alias for `splines::ns()`.) Like `poly()` and `ns()`, the `_M` suffix functions do *NOT* include an intercept column.

**Value**

The `_M` functions return a model matrix. The `_set` functions return a function with arguments `x` and `n`. The integer `n` specifies which function to use, while `x` is the set of values at which to evaluate that function.

---

 Body\_fat

*Measurements of body-fat percentage and related quantities*


---

### Description

It's hard to measure body fat. This data frame records a direct measure of body-fat percentage along with easier-to-make measurements. The question of interest in collecting the data is whether there is a way to estimate body-fat percentage from the easier-to-measure quantities.

### Usage

```
Body_fat
```

### Format

An object of class `spec_tbl_df` (inherits from `tbl_df`, `tbl`, `data.frame`) with 252 rows and 14 columns.

---

 box\_set

*Evenly spaced samples across a one- or two-dim domain*


---

### Description

This function breaks up a domain in 1- or 2- dimensions into evenly spaced samples. It returns a data frame of the position of the samples, each of which can be considered to correspond to a Riemann bin for the purposes of integration.

### Usage

```
box_set(tilde = NULL, domain, n = 10, sum = FALSE, dx = NULL)
```

### Arguments

<code>tilde</code>	A tilde expression specifying the function to be evaluated on the domain.
<code>domain</code>	Either a one- or two-dimensional domain in the same format as for <code>slice_plot()</code> or <code>contour_plot()</code> , or a data frame with two columns specifying the coordinates of a polygon defining the area.
<code>n</code>	the number of divisions along each of the x- and y-directions. Can be a vector of length 2 giving different numbers for the x and for the y directions.
<code>sum</code>	If TRUE carry out the integral and return the numerical result.
<code>dx</code>	An alternative way of specifying the box size directly. Same for all dimensions.

**Value**

By default, a data frame listing the location of the samples, the `.output` value of the given function at those samples, the spatial extent of the samples (that is, `dx` for one-dimension or `dx` and `dy` for two dimensions. There is also a `dA` giving the `dx` or `dx*dy` depending on the dimension). If `sum=TRUE`, then returns the sum of `.output * dA`, that is, the estimate of the integral of the function over the domain.

**Examples**

```
box_set(x*y ~ x & y, domain(x=0:1, y=0:1), n = 4)
# approximation to the variance of a uniform [0,1] distribution
box_set((x-.5)^2 ~ x, domain(x=0:1), n=100, sum=TRUE)
# a polygon
poly <- tibble(x = c(1:9, 8:1), y = c(1, 2*(5:3), 2, -1, 17, 9, 8, 2:9))
boxes <- box_set(1 ~ x & y, poly, dx = 1)
gf_polygon(y ~ x, data = poly, color="blue", fill="blue", alpha=0.2) %>%
  gf_rect((y - dy/3) + (y + dy/3) ~ (x - dx/3) + (x + dx/3),
    data = boxes)
# area inside polygon
box_set(1 ~ x & y, poly, n=100)
```

---

Cello

*Short recordings of a cello and a violin*


---

**Description**

Only one note is involved.

- Cello: a complete note
- Violin: a complete note
- Cello\_seg: a brief snippet of sound after the initial transient
- Violin\_seg: as in Cello\_seg

**Usage**

Cello

Cello\_seg

Violin

Violin\_seg

**Format**

Data frames (for convenience) with many rows and these variables:

- `y` the amplitude of the sound wave (arbitrary units)
- `t` the time of the sample in seconds.

An object of class `tbl_df` (inherits from `tbl`, `data.frame`) with 3616 rows and 2 columns.

An object of class `tbl_df` (inherits from `tbl`, `data.frame`) with 43776 rows and 2 columns.

An object of class `tbl_df` (inherits from `tbl`, `data.frame`) with 3638 rows and 2 columns.

---

Chirps	<i>Cricket chirp rate and temperature</i>
--------	---

---

**Description**

The rate at which crickets chirp is related to the ambient temperature. These data are motivated and described at [https://www.globe.gov/explore-science/scientists-blog/archived-posts/sciblog/index.html\\_p=45.html](https://www.globe.gov/explore-science/scientists-blog/archived-posts/sciblog/index.html_p=45.html)

**Usage**

```
data(Chirps)
```

**Format**

A data frame with 55 rows

- `date` Which year the estimate is for.
- `time` in 24-hour format
- `chirps` the number of chirps per 15 seconds
- `temperature` in degrees F

---

contour_plot	<i>Contour plots of functions of two variables</i>
--------------	--

---

**Description**

Creates a ggplot2-compatible contour plot of a function of two variables.

**Usage**

```

contour_plot(
  ...,
  npts = 100,
  labels = TRUE,
  filled = TRUE,
  contours_at = NULL,
  n_contours = 10,
  n_fill = 50,
  alpha = 1,
  fill_alpha = 0.1,
  label_alpha = 1,
  label_placement = 0.5,
  contour_color = "blue",
  label_color = contour_color,
  skip = 1,
  guide = FALSE,
  guide_title = "output",
  color_scale = scale_color_viridis_c(),
  fill_scale = scale_fill_viridis_d()
)

```

**Arguments**

...	Canonical first "three" arguments: [Previous layer], tilde expression, <a href="#">domain</a>
npts	Integer number of points on each axis at which to evaluate the function
labels	Logical flag: label the contours
filled	Logical flag: fill between the contours
contours_at	Vector of numbers. Contours will be drawn at these levels of the output.
n_contours	hint at number of contours to show
n_fill	Integer number of points for calculating fill
alpha	Transparency of contours in [0-1]
fill_alpha	Transparency of fill
label_alpha	Likewise, for contour labels
label_placement	A number, between 0 and 1, suggesting where to place the contour labels. Default: 0.5. This can be useful when there are two contour layers in one plot.
contour_color	Set to a string, e.g. "blue" for contours to be drawn in a fixed color
label_color	Defaults to contour_color. Can be set to a
skip	Small integer. Skip this many contours between labeled contours
guide	Logical flag, whether to show a color guide.
guide_title	Character string for title of guide (if any)
color_scale	See <a href="#">ggplot2 color scales</a> .
fill_scale	See <a href="#">ggplot2 fill scales</a>

**Value**

ggplot2 graphics layers

**Examples**

```
## Not run:
contour_plot(sin(0.2*x*y) ~ x & y, domain(x=-10.3:4.5, y=2:7.5))
contour_plot(x + y ~ x & y)
contour_plot(sin(0.2*x*y)) # but better to use tilde expression

## End(Not run)
```

---

Covid\_US

*COVID data from the first half of the pandemic*

---

**Description**

For documentation, see <https://covid19datahub.io/articles/doc/data.html>

**Usage**

Covid\_US

**Format**

An object of class `grouped_df` (inherits from `tbl_df`, `tbl`, `data.frame`) with 575 rows and 36 columns.

**Details**

See Guidotti, E., Ardia, D., (2020), "COVID-19 Data Hub", Journal of Open Source Software 5(51):2376, doi: 10.21105/joss.02376.

---

CPUs

*Characteristics of computer central processing unit chips over the decades*

---

**Description**

Since single-chip central processing units were introduced in the early 1970s, performance has increased exponentially. Around 1965, Gordon E. Moore, a co-founder of the integrated circuit giant Intel, postulated that capacity of chips, measured as the number of transistors, doubles approximately every two years. Known as Moore's Law, this pattern has held true for decades.

**Usage**

```
data(CPUs)
```

**Format**

A data frame with 128 cases, each a computer CPU.

- processor Model name/number of the processor
- transistors Number of transistors
- year Year of market introduction of the CPU
- designer The company that introduced the CPU
- process The width of circuit elements, in nm.
- area Area of the chip in mm-squared.
- density The number of transistors per unit area, simply transistors/area.

**Source**

Dr. Joe Eichholz

---

create_num_antiD	<i>Create a numerical anti-derivative function which can be called with one or many values of the w.r.t. input</i>
------------------	--

---

**Description**

This will typically be called directly from antiD() when an integral can't be handled symbolically.

**Usage**

```
create_num_antiD(tilde, ..., lower = NULL, .tol = 1e-04)
```

**Arguments**

tilde	Tilde expression for the function to be anti-differentiated. Right-hand side will be the w.r.t. variable
...	arguments and parameters to the function described by tilde
lower	Optional lower bound of integration. Useful to avoid domain problems with the function being integrated, but not generally needed.
.tol	Numerical tolerance for the integration

**Value**

a function with the w.r.t. variable as the first argument. The function is a wrapper around numerical integration routines.

**Description**

Operators for computing derivatives and anti-derivatives as functions.

**Usage**

```
D(tilde, ...)
```

```
antiD(tilde, ..., lower.bound = 0, force.numeric = FALSE, .tol = 1e-04)
```

**Arguments**

<code>tilde</code>	A tilde expression. The right side of a formula specifies the variable(s) with which to carry out the integration or differentiation. On the left side should be an expression or a function that returns a numerical vector of the same length as its argument. The expression can contain unbound variables. Functions will be differentiated as if the formula $f(x) \sim x$ were specified but with $x$ replaced by the first argument of $f$ .
<code>...</code>	Default values to be given to unbound variables in the expression <code>expr</code> . See examples.#' Note that in creating anti-derivative functions, default values of "from" and "to" can be assigned. They are to be written with the name of the variable as a prefix, e.g. <code>y.from</code> .
<code>lower.bound</code>	for numerical integration only, the lower bound used
<code>force.numeric</code>	If TRUE, a numerical integral is performed even when a symbolic integral is available.
<code>.tol</code>	Tolerance for numerical integration. Most users do not need this.

**Details**

`D()` attempts to find a symbolic derivative for simple expressions, but will provide a function that is a numerical derivative if the attempt at symbolic differentiation is unsuccessful. The symbolic derivative can be of any order (although the expression may become unmanageably complex). The numerical derivative is limited to first or second-order partial derivatives (including mixed partials). `antiD()` will attempt simple symbolic integration but if it fails it will return a numerically-based anti-derivative.

`antiD()` returns a function with the same arguments as the expression passed to it. The returned function is the anti-derivative of the expression, e.g., `antiD(f(x)~x) -> F(x)`. To calculate the integral of  $f(x)$ , use  $F(\text{to}) - F(\text{from})$ .

**Value**

For derivatives, the return value is a function of the variable(s) of differentiation, as well as any other symbols used in the expression. Thus,  $D(A*x^2 + B*y \sim x + y)$  will compute the mixed partial with respect to  $x$  then  $y$  (that is,  $\frac{d^2 f}{dy dx}$ ). The returned value will be a function of  $x$  and  $y$ , as well as  $A$  and  $B$ . In evaluating the returned function, it's best to use the named form of arguments, to ensure the order is correct.

a function of the same arguments as the original expression with a constant of integration set to zero by default, named "C", "D", ... depending on the first such letter not otherwise in the argument list.

**Examples**

```
D(sin(t) ~ t)
D(A*sin(t) ~ t )
D(A*sin(2*pi*t/P) ~ t, A=2, P=10) # default values for parameters.
f <- D(A*x^3 ~ x + x, A=1) # 2nd order partial -- note, it's a function of x
f(x=2)
f(x=2,A=10) # override default value of parameter A
g <- D(f(x=t, A=1)^2 ~ t) # note: it's a function of t
g(t=1)
gg <- D(f(x=t, A=B)^2 ~ t, B=10) # note: it's a function of t and B
gg(t=1)
gg(t=1, B=100)
f <- makeFun(x^2~x)
D(f(cos(z))~z) #will look in user functions also
antiD( a*x^2 ~ x, a = 3)
G <- antiD( A/x~x ) # there will be an unbound parameter in G()
G(2, A=1) # Need to bound parameter. G(2) will produce an error.
F <- antiD( A*exp(-k*t^2 ) ~ t, A=1, k=0.1)
F(t=Inf)
one = makeFun(1 ~ x)
by.x = antiD(one(x) ~ x)
by.xy = antiD(by.x(sqrt(1-y^2)) ~ y)
4 * by.xy(y = 1) # area of quarter circle
```

**Description**

The *MOSAIC Calculus* course includes a block on linear algebra. As part of this block, we want to cover creating model matrices and evaluating the model constructed from them. One way to do this is to introduce `lm()` and the domain specific language for specifying model terms. However, that introduces oddities. For instance, `lm(mpg ~ hp + hp^2, data = mtcars)` does NOT add the quadratic term to the model matrix. Also, `lm()` doesn't produce a residual or the model vector (ahem ... the fitted values) in the form of vectors. That's fine if you're teaching statistical modeling, but in the *MOSAIC Calculus* linear algebra block we are not teaching statistics, but the mathematical pre-requisites to understanding statistics.

**Usage**

```
df2matrix(..., data = NULL)
```

**Arguments**

... Expressions, written in terms of the variable names in the data frame, that are to be collected into the model matrix.

data The data frame from which to which the variable names will be bound.

**Details**

Specifically for *MOSAIC Calculus*, we have added this `df2matrix()` function. It serves much the same purpose as `cbind()`, that is, collecting vectors into a matrix. But it has two additional features:

1. It has a `data=` argument so that it can refer to a data frame.
2. It names the columns of the matrix with the code that was used to create each column.

The consequence of (2) is that the `x` vector produced by `qr.solve()` will have the same names as the matrix. That helps in interpreting `x`. But those names can also be used by `makeFun()` to generate a function from `x`.

1 is a good form in which to write the intercept term.

**Value**

a matrix

**Examples**

```
A <- df2matrix(1, disp, log(hp), sin(cyl)*sqrt(hp), data = mtcars)
b <- df2matrix(mpg, data = mtcars)
x <- qr.solve(A, b)
f <- makeFun(x)
f(hp=3, disp=2, cyl=4)
```

---

domain

*Specify a domain over which a function is to be graphed*

---

**Description**

`domain()` is used with `slice_plot()`, `contour_plot()`, or `interactive_plot()` to describe the plotting region. There is a standard syntax for `domain()` (see the first example) but there are also shortcuts. In the shorthand syntaxes, you can but don't have to specify the name of the input. If it's not specified, the plotting programs will try to do something sensible. But better to specify the names explicitly.

**Usage**

```
domain(...)
```

**Arguments**

```
...          One or more expression of the form x = -5:5
```

**Details**

The colon operator is masked so that, for instance, `x = 0.5:1.3` literally means "0.5 to 1.3", and not just 0.5 as the base colon operator would give.

**Value**

a list with one component for each element in ...

**Examples**

```
## Not run:
contour_plot(x / y ~ x + y, domain(x=-5:5, y=1:4))
slice_plot(x^2 ~ x, domain(x = 2.5:4.2)) # overrides colon operator
slice_plot(x^2 ~ x, domain(x = c(2.5, 4.2)))
slice_plot(x^2 ~ x, domain(x = 1 %pm% 0.5))

## End(Not run)
```

---

doodle\_fun

---

*Construct a random function that is smooth*


---

**Description**

Sometimes it's nice to have a simple, smooth but curving function to illustrate some mathematical point or to support an exercise. `doodle_fun()` generates such functions. The function support tends to be in -10 to 10. For the mathematically adept ... the functions are linear combinations of gaussians with randomly selected centers, widths, and random coefficients. The `mosaic::rfun()` function is very similar.

**Usage**

```
doodle_fun(vars = ~x & y, seed = NULL, n = 0)
```

**Arguments**

```
vars          A right-sided tilde expression (like ~ x & y) that indicates how many inputs there
              should be and their names.
seed          An integer setting the seed for random number generation.
n            A positive integer. If non-zero, this is a hint about how many local maxes and
              mins there should be. Just a hint.
```

**Examples**

```
# Two functions of two variables. They are different because the seeds are different.
f <- doodle_fun(~ x & y, seed = 101)
g <- doodle_fun(~ z & u, seed = 121)
```

EbolaAll

*Case numbers in an Ebola outbreak in 2014***Description**

In December 2013, an 18-month-old boy from a village in Guinea suffered fatal diarrhea. Over the next months a broader outbreak was discovered, and in mid-March 2014, the Pasteur Institute in France confirmed the illness as Ebola-Virus Disease caused by the Zaire ebolavirus. Although the outbreak was first recognized in Guinea, it eventually encompassed Liberia and Sierra Leone as well. By July 2014, the outbreak spread to the capitals of all three countries.

**Usage**

EbolaAll

EbolaGuinea

**Format**

A dataframe with 182 rows. Each row is a daily report during a period of over 18 months during 2014 and 2015. Each report gives the number of new cases and disease-related deaths since the last report in each or three countries: Sierra Leon, Liberia, and Guinea. These values have been calculated from the raw, cumulative data. The data have been scrubbed to remove obvious errors.

- Date: Date when the World Health Organization issued the report
- Gcases: Number of new cases in Guinea
- Gdeaths: Number of new deaths in Guinea
- Lcases: Number of new cases in Liberia
- Ldeaths: Number of new deaths in Liberia
- SLcases: Number of new cases in Sierra Leone
- SLdeaths: Number of new deaths in Sierra Leone
- TotCases: Cumulative number of cases across all three countries
- TotDeaths: Cumulative number of deaths across all three countries Added in EbolaGuinea
- Days: When the report was issued in terms of a count of days from the initial report.
- G7Rcases: Number of new cases in Guinea averaged across 7 reports
- G7Rdeaths: Number of new deaths in Guinea averaged across 7 reports

An object of class `spec_tbl_df` (inherits from `tbl_df`, `tbl`, `data.frame`) with 182 rows and 6 columns.

**Details**

- EbolaAll gives daily reports for Guinea, Liberia, and Sierra Leone.
- EbolaGuinea contains just the Guinea data, but adds columns containing a 7-day moving average.

**Source**

US Centers for Disease Control (CDC).

---

Effective_oxygen	<i>Effective amount of oxygen available at different altitudes</i>
------------------	--

---

**Description**

Air pressure changes with altitude and therefore the amount of oxygen available changes as well.

**Usage**

```
data(Effective_oxygen)
```

**Format**

A data frame with 30 rows

- altitude‘ In feet above sea level
- oxygen‘ Effective oxygen as a proportion of ai-

**Source**

<https://wildsafe.org/resources/ask-the-experts/altitude-safety-101/oxygen-levels/>

---

Engines	<i>Characteristics of various internal combustion engines</i>
---------	---

---

**Description**

Internal combustion engines have been built for a variety of purposes ranging from propelling small model airplanes to powering trucks to driving the propellers of giant ships. This data frame gives the characteristics of a wide size range of such engines.

**Usage**

```
data(Engines)
```

**Format**

A data frame with 39 cases, each of which is an internal combustion engine, with observations on the following variables.

- mass In pounds
- ncyliner Number of cylinders
- stroke The length of the stroke made by a piston
- strokes Whether the engine cycle is 2-stroke or 4-stroke.
- displacement In cc
- bore Diameter of the piston
- BHP The power generated by the engine. One BHP is the same as 745.7 Watts.
- RPM The speed, in revolutions per second, at which the engine generates the listed BHP.

**Source**

McMahon, Thomas A., and John Tyler Bonner. On Size and Life. New York: Scientific American Library, 1983. pp. 60-61

---

<code>first_three_args</code>	<i>Handle the first three arguments of graphics functions</i>
-------------------------------	---

---

**Description**

This function is intended for package developers, not end-users. Canonically, `mosaicCalc` functions that produce layerable graphics have three initial arguments in a specific order: (1) a previous `gg` layer, (2) a tilde expression, and (3) a domain. But either (1) or (3) can be missing. `first_three_args()` translates a leading `...` argument into the list of the canonical three initial arguments, returning them as components of a list. In addition, there may be additional arguments in `...` that specify other aspects of the plot, e.g. `color`.

**Usage**

```
first_three_args(..., two_tildes = FALSE)
```

**Arguments**

<code>...</code>	unnamed arguments to be translated into a list with the three canonical arguments and any other arguments not named explicitly in the parent function definition.
<code>two_tildes</code>	if TRUE then look for the first FOUR arguments, the middle two of which will be tilde expressions.

**Details**

In constructing a mosaicCalc graphics layer, the function (e.g. `slice_plot()` or `contour_plot()`) would have ... as its first argument. Intercept that ... with `first_three_args()` to extract the first three canonical arguments as components `gg`, `tilde`, and `domain` of a list. Any remaining arguments in ... will be placed in the `dots` component.

---

fitSpline	<i>Find zeros of a function</i>
-----------	---------------------------------

---

**Description**

This is defined in the `mosaic` package: See [fitSpline](#)

---

Fly_ball	<i>Trajectory of a fly ball in baseball</i>
----------	---

---

**Description**

The trajectory of a fly ball as calculated by a sophisticated model involving drag, spin, and such. The physics and mathematics are described at <http://baseball.physics.illinois.edu/TrajectoryAnalysis.pdf>

**Usage**

```
Fly_ball
```

**Format**

A data frame with 552 rows.

- `t` - (seconds). Runs from 0 to 5.51 s
- `y` - horizontal position (feet)
- `z` - vertical position (feet)

**Details**

The ball has a mass of 5.125 ounces and a circumference of 9.125 inches The speed off the bat was 103 mph at a launch angle of 27.5 degrees from the horizontal. The ball was hit with backspin at 2500 rpm by a right-handed batter. Ambient temperature 70 deg F with a relative humidity of 50% and a barometric pressure of 29.92 inHg. The field was at an elevation of 15 feet.

The position of the ball was recorded every 0.01 seconds, for 5.5 seconds altogether. If you prefer to work with continuous functions of time, you can construct them. See the examples.

**Source**

<http://baseball.physics.illinois.edu/TrajectoryCalculator-new-3D.xlsx>

**Examples**

```
# Constructing continuous functions of time
ball_z <- mosaic::connector(z ~ t, data = Fly_ball)
ball_y <- mosaic::connector(y ~ t, data = Fly_ball)
```

---

gradient\_plot

*Plot a vector field*

---

**Description**

Plot a vector field

**Usage**

```
gradient_plot(..., npts = 20, color = "black", alpha = 0.5, transform = sqrt)
```

```
vectorfield_plot(
  ...,
  npts = 20,
  color = "black",
  alpha = 0.5,
  transform = sqrt,
  stretch = 1,
  env = NULL
)
```

**Arguments**

...	(Optional: a previous graphics layer), tilde expressions for horiz and vertical component of vectors, a domain (if not inherited from the graphics layer). The tilde expressions should be in the same style as for <code>makeODE()</code> . See details.
npts	number of arrows to draw in one row or column of the field.
color	character string specifying the color of the arrows.
alpha	transparency of the arrows
transform	controls the relative length of the arrows. See details.
stretch	Stretch the arrows by this factor (default: 1).
env	Not for end-users. Handles details of where things are defined.

## Details

There will be one tilde expression for the horizontal component of the vector and another tilde expression for the vertical component. Suppose the horizontal axis is called  $u$  and the vertical axis is called  $v$ , as would be established by a bounds specification like `domain(u=-1:1, v=-1:1)`. Then the horizontal tilde expression **must** have a left side called  $u \sim$ . Similarly, the vertical tilde expression will have a left side called  $v \sim$ . On the right side of the tilde expressions will go the formulas for the respective components of the vectors, e.g.  $u \sim \sin(u-v)$  and  $v \sim v*u^2$ .

Typically, the length of the arrows is not meaningful in the units of the horizontal or vertical axis. For instance, in a gradient plot of  $f(x,y)$ , the axis is in units of  $x$ , but the gradient component has units of  $f(x,y)/x$ . Similarly for the flow of a differential equation. Nonetheless, the relative lengths of the arrows, one to another, does have meaning. In drawing the vector field, the arrows are scaled so that the longest one barely avoids contact with it's neighbors. This natural scaling has the disadvantage that it can be hard to discern the lengths of the shortest arrows, which often are near zero (as with the gradient near the argmax or argmin, or near a fixed point of a differential equation flow field). By default, the relative lengths of the arrow are transformed by `sqrt`, to make the long arrows shorter and therefore enable the sort arrows to be drawn somewhat longer. If you want the natural scaling instead, use `transform=I`. Or you might want to make the arrows even more similar in length. Then use, for instance `transform=function(L) L^0.1`

Occasionally, you may wish to display the arrows only within a *valid* region of the rectangular state space. To indicate invalidity, the functions in the tilde expressions should return NA for invalid state values.

## Value

ggplot2 graphics layers

## Examples

```
gradient_plot(x * sin(y) ~ x & y, domain(x=-1:1, y=-1:1), transform=I)
vectorfield_plot(x ~ -y, y ~ x, domain(x=-1:1, y=-1:1))
gf_label(0 ~ 0, label="center", color="red") %>%
vectorfield_plot(x ~ -y, y ~ x, domain(x=-1:1, y=-1:1), transform=function(x) x^0.2 )
vectorfield_plot(u ~ sin(u-v), v ~ v*u^2, domain(u=0:1, v=-1:1))
```

## Description

A "heating degree day" is a measure of weather coldness. It's defined to be the difference between the outdoor ambient temperature and 65 degrees F, but has a value of zero when the ambient temperature is above 65 degrees. This difference is averaged over time and multiplied by the number of days in the time period covered. The heating degree day is often used as a measure of the demand for domestic heating in a locale.

**Usage**

```
data(HDD_Minneapolis)
```

**Format**

A data frame `HDD_Minneapolis` with 1412 rows and 4 variables:

- `year` the year
- `month` the month
- `hdd` the number of heating degree days for that period.
- `loc` the location at which the temperature was measured. In the early years, this was downtown Minneapolis. Later, the site was moved to the Minneapolis/Saint-Paul International Airport.

**Details**

These data report monthly heating degree days. For teaching purposes, the data give an extreme example of how a relationship (`hdd` vs `year`) can be revealed by including a covariate (`month`). Although interest focusses on the change in temperature over the century the data cover, there is such regular seasonal variation that no systematic trend over the years is evident unless `month` is taken into account.

**Examples**

```
mod_1 <- lm(hdd ~ year, data = HDD_Minneapolis)
mod_2 <- lm(hdd ~ year + month, data = HDD_Minneapolis)
```

---

Home\_utilities

*Gas and electricity usage by a home in St. Paul, MN*

---

**Description**

Gas and electricity usage by a home in St. Paul, MN

**Usage**

```
data(Home_utilities)
```

**Format**

A data.frame object with one row for each month

- `month` - the month covered by the bill as a number
- `day` - the terminating day of the bill
- `year` - the year in which the month fell
- `temp` - average temperature over the entire month
- `kwh` - electricity usage in kilowatt-hours

- ccf - natural gas usage in cubic feet
- thermsPerDay - another measure of natural gas usage
- dur - duration of the period covered by the bill, in days (typically about 30)
- totalbill - dollar amount of the bill
- gasbill: - dollar amount of the natural gas part of the bill
- elecbill - dollar amount of the electricity bill
- generatedKwh - Energy generated by the photovoltaic system
- notes - miscellaneous comments on the month's bill or changes in the house

### Details

A college mathematics professor teaching in St. Paul, Minnesota, USA collected these records of his utility bills starting in 1999. Starting in June 2022, solar photovoltaics were installed on the roof of the house. There are months missing, a few transcription errors, etc. **These data will be updated occasionally, as new bills come in.**

### Source

Personal communication from the homeowner

---

inequality\_constraint *Graphics for constraints*

---

### Description

These functions are intended to annotate contour plots with constraint regions. Inequality constraints are shaded where the constraint is NOT satisfied. Equality constraints are shaded in a small region near where the constraint is satisfied.

### Usage

```
inequality_constraint(  
  previous = NULL,  
  tilde,  
  domain,  
  npts = 100,  
  fill = "blue",  
  alpha = 1  
)
```

```
equality_constraint(  
  previous = NULL,  
  tilde,  
  domain,  
  npts = 100,  
  fill = "blue",  
  alpha = 1  
)
```

**Arguments**

previous	can be ignored by user. It supports the pipe syntax for layering graphics.
tilde	a tilde expression specifying the constraint. For an inequality constraint this should be a logical expression that is TRUE where the constraint is satisfied. For an equality constraint, the left-hand side of the tilde expression should be zero where the constraint IS satisfied.
domain	as in <code>contour_plot()</code> , the domain over which to graph the constraint
npts	a number specifying how finely to divide the domain in each direction. Default is 100, but this gives a discernably pixelated appearance to the shading. 200 or 300 is more appropriate for publication-quality graphics.
fill	the color to use for shading
alpha	the opacity of the shading

**Value**

ggplot2 graphics layers

**Examples**

```
inequality_constraint(x + y > 2 ~ y + x, domain(y=0:3, x=0:2))
equality_constraint(x + y - 2 ~ y + x, domain(y=0:3, x=0:2), npts=200, alpha=.3, fill="red")
```

---

infer\_RHS

*Utilities for formulas and graphics arguments*


---

**Description**

`infer_RHS` turns a one-sided formula into a two-sided formula suitable for `makeFun()`. `formals_from_expression` creates a "formals" list for creating a function. The list will have arguments in the canonical order.

**Usage**

```
infer_RHS(ex)

formals_from_expr(ex, others = character(0))
```

**Arguments**

ex	An expression as in <code>quote(x^2)</code> or the left side of a tilde expression.
others	Character string(s) with names of additional arguments to be included in the formals

---

inscribed_circle	<i>Create a data frame for a circle marking the curvature of a function.</i>
------------------	--

---

**Description**

Create a data frame for a circle marking the curvature of a function.

**Usage**

```
inscribed_circle(ftilde, x0 = 0)
```

**Arguments**

ftilde	A tilde expression defining a function
x0	The value for the input at which the curvature is to be calculated.

**Value**

inscribed\_circle() return a data frame with x and y components describing the inscribed circle located at the point  $(x_0, f(x_0))$ . Plot this with `gf_path(y ~ x) curvature_function()` returns a function that gives the curvature of the specified function for any input  $x$ . It's much like `D()` in the way it is used.

---

Integrate	<i>Integrate a function</i>
-----------	-----------------------------

---

**Description**

Calculates the definite integral of a function. That is, the result of the integration will be a number. There can be no free parameters in the function being integrated. (If you want free parameters, use `antiD()`.) In regular R, `Integrate()` can handle integration over up to 3 variables, but only 1 variable in `webr`.

**Usage**

```
Integrate(tilde, domain, ..., tol = 1e-05)
```

**Arguments**

tilde	A tilde expression describing the function.
domain	The domain over which to perform the integration
...	values assigned to free parameters in the tilde expression, e.g. <code>a=1</code>
tol	numerical tolerance for integration, see <code>stats::integrate()</code> .

**Details**

For functions constructed as a spline interpolant, `Integrate()` can handle more segments than `antiD()`. It's reasonable to do up to 2000 segments with `Integrate()`, whereas `antiD()` handles only about 100.

**Examples**

```
Integrate(dnorm(x) ~ x, domain(x=-2:2))
Integrate(dnorm(x, sd=sigma) ~ x, domain(x=-2:2), sigma=2)
Integrate(sqrt(1- x^2) ~ x, domain(x=-1:1)) # area of semi-circle
```

---

integrateODE

*Integrate ordinary differential equations*


---

**Description**

A formula interface to integration of an ODE with respect to "t"

**Usage**

```
integrateODE(...)
```

**Arguments**

... A dynamics object (see `makeODE()`) and/or arguments giving additional formulas for dynamics in other variables, assignments of parameters, assignments of initial conditions, the start and end times of the integration (through `domain()`), and the step size (through `dt=`).

**Details**

The equations must be in first-order form. Each dynamical equation uses a formula interface with the variable name given on the left-hand side of the formula, preceded by a `d`, so use `dx~-k*x` for exponential decay. All parameters (such as `k`) must be assigned numerical values in the argument list. All dynamical variables must be assigned initial conditions in the argument list. The returned value will be a list with one component named after each dynamical variable. The component will be a spline-generated function of `t`.

**Value**

a list with splined function of time for each dynamical variable

**Examples**

```

soln = integrateODE(dx~r*x*(1-x/k), k=10, r=.5, domain(t=0:20), x=1)
soln$x(10)
soln$x(30) # outside the time interval for integration
traj_plot(x(t)~t, soln, domain(t=0:10))
soln2 = integrateODE(dx~y, dy~-x, x=1, y=0, domain(t=0:10))
traj_plot(y(t)~t, soln2)
SIR <- makeODE(dS~ -a*S*I, dI ~ a*S*I - b*I, a=0.0026, b=.5, S=762, I=1)
epi = integrateODE(SIR, domain(t=0:20))
traj_plot(S(t) ~ t, epi)
traj_plot(t ~ t, epi)

```

---

is_in_domain	<i>check whether a value is in a domain</i>
--------------	---

---

**Description**

A convenience function to see if a value is inside (or on the boundaries of) a domain or a set of domain segments. In the case of multiple segments, the check is whether val is within any of them.

**Usage**

```
is_in_domain(val, domain, ...)
```

**Arguments**

val	the values to be checked
domain	the domain
...	additional domains

**Examples**

```

is_in_domain(1:10, domain(x=5.5:8.5), domain(x=1:2))
is_in_domain(mtcars, domain(mpg=15:20, wt=2:4.5), domain(mpg=25:28))

```

---

Iterate	<i>Iterate a function on an initial condition</i>
---------	---

---

**Description**

Iterates a function a specified number of times on an initial condition.

**Usage**

```
Iterate(f = NULL, A = NULL, x0 = 0, n = 10, fargs = list())
```

**Arguments**

f	a function of one or more state variables which returns a vector containing those same state variables. Parameters to <code>f()</code> , if any, should be named and at the end of the argument list. State variables should <b>not</b> have default values.
A	As an alternative to <code>f</code> you can give a square matrix and iteration will be done on the system $x[n+1] = A x[n]$ . In lieu of a square matrix you can also give a vector to be rendered into a square matrix rowwise.
<code>x0</code>	a vector with the numerical initial condition. There should be 1 component in <code>x0</code> for each of the state variables in <code>f()</code> .
n	an integer specifying the number of iterations
fargs	list containing values for numerical parameters to the function <code>f()</code>

**Details**

The function `f` can take one or more arguments. The first of these should represent the *state* of the dynamical system, e.g. `x`, or `x` and `y`, etc. At the end of the argument list to `f` can come named parameters. The length of the initial condition `x0` must match the number of state arguments. Numerical values for parameters (if any) must be provided in the `...` slot.

**Value**

A data frame with a column `.i` listing the iteration number and columns for each component of the initial condition. There will be `n+1` rows, the first for the initial condition and the remaining for the `n` iterations.

**Examples**

```
Iterate(function(x, mu=3.5) mu*x*(1-x), x0=.232, n=10, list(mu=4)) # chaos
Iterate(function(x, y) c(x+y, x-y), x0 = c(1,1), n=10)
Iterate(function(x, y) c(x+y, x), x0=c(1,0), n=10) # fibonacci
Iterate(A = cbind(rbind(1, 1), rbind(1, 0)), x0=c(1,0), n=5) # fibonacci described by a matrix
```

---

Kepler

*Kepler's calculation of the position of Mars*


---

**Description**

Astronomer **Johannes Kepler** (1571-1630) famously measured the position of the planet Mars. His interpretation of this data led substantially to Newton's theory of universal gravity. This data frame gives Kepler's measurements as a function of time, together with a modern reconstruction of the "actual" position of Mars at the time.

**Usage**

```
data(Kepler)
```

**Format**

A data frame with 28 rows and these variables:

- `time` - Interval in Earth days from 8:15am Greenwich time on 9 March 1584.
- `kepler.radius` - The radius of the orbit in AU (astronomical units. 1 AU is 93 million miles)
- `kepler.angle` - The "true anomaly" measured in radians
- `actual.radius` - Modern calculation of the above
- `actual.angle` - Modern calculation of the above

**Details**

The raw measurements (not included here) that Kepler used in his calculation were made by Tycho Brahe (1546-1601). Those raw measurements were of the angle of Mars with respect to Earth. Kepler estimated the orbital period of Mars to be 687 Earth days. (The current accepted value is 686.980 days.) Knowing the period, Kepler could find pairs of Earth days separated by multiples of the period. In each pair, the Earth would be in a different position, but Mars would be in the same position. Thus the distance of Mars from Earth could be estimated by triangulation.

The angle was not directly measured for each occasion. Instead, knowing the radius versus time Kepler was able to discern when Mars was at its greatest and closest distance to the Sun. The angle tells where Mars is along its orbit. An angle of 0 is the position when Mars is closest to the Sun. An angle of 3.14 is when Mars is farthest from the Sun.

**Source**

Drawn from from McLaughlin, Michael P. ( 1999 ) "A Tutorial on **Mathematical Modelling**" p. 21-23.

**References**

See <https://faculty.uca.edu/saustin/3110/mars.pdf> for a useful description of the estimation process.

---

M2014F

*Mortality versus age for females in the US in 2014*


---

**Description**

These data are from the US Social Security Administration.

**Usage**

M2014F

**Format**

An object of class `tbl_df` (inherits from `tbl`, `data.frame`) with 120 rows and 5 columns.

**Details**

- pmort the probability of a person of the given age dying in 2014
- nliving traces a hypothetical population of 100,000 people through the mortality at each age.
- died for the hypothetical population the number of people at that age who survived to the age and then died at that age. Source: <https://www.ssa.gov/oact/HistEst/PerLifeTables/2017/PerLifeTables2017.html>

---

makeODE	<i>Create a dynamics object for use in integrateODE() and the ODE graphics</i>
---------	--

---

**Description**

An ODE object consists of some dynamics, initial conditions, parameter values, time domain, etc.

**Usage**

```
makeODE(...)
```

**Arguments**

... The components of an ODE and/or a set of other ODE objects.  
By default, the time step is set to  $dt = 0.01$ . Change it with  $dt =$  value.

**Value**

a list containing various functions and values specifying the ODE.

**Examples**

```
SIR <- makeODE(dS~ -a*S*I, dI ~ a*S*I - b*I, a=0.0026, b=.5, S=762, I=1)
soln <- integrateODE(SIR, domain(t=0:20))
```

---

numD	<i>Numerical Derivatives</i>
------	------------------------------

---

**Description**

Constructs the numerical derivatives of mathematical expressions

**Usage**

```
numD(tilde, ..., .h = NULL)
```

**Arguments**

tilde	a mathematical expression (see examples and <a href="#">plotFun</a> )
...	additional parameters, typically default values for mathematical parameters
.h	numerical step size to enforce.

**Details**

Uses a simple finite-difference scheme to evaluate the derivative. The function created will not contain a formula for the derivative. Instead, the original function is stored at the time the derivative is constructed and that original function is re-evaluated at the finitely-spaced points of an interval. If you redefine the original function, that won't affect any derivatives that were already defined from it. Numerical derivatives, particularly high-order ones, are unstable. The finite-difference parameter `.h` is set, by default, to give reasonable results for first- and second-order derivatives. It's tweaked a bit so that taking a second derivative by differentiating a first derivative will give reasonably accurate results. But, if taking a second derivative, much better to do it in one step to preserve numerical accuracy.

**Value**

a function implementing the derivative as a finite-difference approximation. This has a second argument, `.h`, that allow the finite-difference to be set when evaluating the function. The default values are set for reasonable numerical precision with the pattern-book functions.

**Note**

WARNING: In the expressions, do not use variable names beginning with a dot, particularly `.f` or `.h`

**Author(s)**

Daniel Kaplan (<[kaplan@macalester.edu](mailto:kaplan@macalester.edu)>)

**Examples**

```
g = numD( a*x^2 + x*y ~ x, a=1)
g(x=2,y=10)
gg = numD( a*x^2 + x*y ~ x&x, a=1)
gg(x=2,y=10)
ggg = numD( a*x^2 + x*y ~ x&y, a=1)
ggg(x=2,y=10)
h = numD( g(x=x,y=y,a=a) ~ y, a=1)
h(x=2,y=10)
f = numD( sin(x)~x)
# slice_plot( f(3,.h=hlim)~h, domain(h=.0000001 :000001)) %>% gf_hline(yintercept = cos(3))
```

---

 PE\_fun1

*Potential energy functions used as examples in MOSAIC Calculus.*


---

**Description**

These appear in the "Optimization & constraint" chapter.

**Usage**

```
PE_fun1(x1, y1)
```

```
PE_fun2(x1, y1, x2, y2, x3, y3)
```

**Arguments**

```
x1, y1, x2, y2, x3, y3
```

coordinates of the masses.

---

 Picket

*Creates a "picket fence" of points for illustrating numerical integration*


---

**Description**

Creates a "picket fence" of points for illustrating numerical integration

**Usage**

```
Picket(
  domain,
  h = 0.1,
  method = c("left", "right", "center", "trapezoid", "gauss")
)
```

**Arguments**

domain	domain of integration as used in <code>slice_plot()</code> , <code>Integrate()</code> , and the similar <code>mosaicCalc</code> functions
h	number giving the width between pickets. Could also have been called <code>dt</code> or <code>dx</code> , and so on.
method	determines the weights for each element in the picket

---

Planet_solar	<i>Solar irradiance of the planets</i>
--------------	--

---

**Description**

Brightness of the sun at the various planets

**Usage**

```
data(Planet_solar)
```

**Format**

A data frame with 9 rows

- planet Name of the planet (or planetoid)
- distance distance from the sun in millions of km
- irradiance solar brightness (in what units?)

---

Planets	<i>NASA data on planets</i>
---------	-----------------------------

---

**Description**

NASA data on planets

**Usage**

```
data(Planets)
```

**Format**

A data frame with 10 rows and these variables:

- planet name of the planet
- mass in  $10^{24}$  kg
- diameter in km
- density in  $\text{kg/m}^3$
- gravity in  $\text{m/s}^2$
- escape\_velocity in km/s
- rotational\_period in hours
- day\_length in hours
- distance\_from\_sun in  $10^6$  km

- perihelion in  $10^6$  km
- aphelion in  $10^6$  km
- orbital\_period in days
- orbital\_velocity in km/s
- orbital\_inclination in degrees
- orbital\_eccentricity
- obliquity\_to\_orbit in degrees
- mean\_temperature in C
- surface\_pressure in bars
- n\_moons a count
- ring\_system does the planet have a ring system
- global\_magnetic\_field does the planet have a magnetic field

### Details

Two of the rows, MOON and PLUTO, are not considered planets. The orbital parameters of the MOON are with respect to EARTH. Others are with respect to the Sun. Negative rotation period denotes a rotation opposite that of most of the other planets.

### Source

<https://nssdc.gsfc.nasa.gov/planetary/factsheet/>

---

plotFun	<i>Plot functions of one and two variables using lattice system</i>
---------	---

---

### Description

This is defined in the mosaic package: See [plotFun](#)

---

qspliner	<i>Create a quadratic spline (inefficiently)</i>
----------	--

---

### Description

A handmade function to construct quadratic splines.

### Usage

```
qspliner(tilde, data, free = 0)
```

**Arguments**

tilde	A tilde expression of the form $y \sim x$ specifying the output variable (on the LHS) and the input variable (on the RHS). Together with data, these (x,y) pairs will be the knots for the spline.
data	A data frame containing the variables in tilde.
free	A number specifying the slope of the output function at the last knot. Default is 0: that is, flat.

**Details**

Unless you have a good reason otherwise, you should be using `spliner()`, which generates cubic splines, rather than `qspliner()`. `qspliner()` is intended only for demonstration purposes.

**Value**

a function suitable for, for instance, graphing or optimizing

**Examples**

```
Pts <- tibble(x = seq(-4,4, by=.7), y = dnorm(x))
f <- qspliner(y ~ x, data = Pts)
slice_plot(dnorm(x) ~ x, domain(x=-4:4)) %>%
  slice_plot(f(x) ~ x, color= "blue") %>%
  gf_point(y ~ x, data = Pts, color = "orange", size=4, alpha=0.3) %>%
  gf_lims(y= c(NA,.5))
```

---

rfun

*Generate a "natural looking" function of one or multiple variables*


---

**Description**

This is defined in the `mosaic` package: See [rfun](#).

---

RI\_tide

*Tide levels from the US NOAA*


---

**Description**

- `RI_tide`: For Providence, RI, Minute-by-minute tide data during April 1-5 2010.
- `Anchorage_tide`: For Anchorage, AK, Every six-minute data during 2018

**Usage**

```
RI_tide
```

**Format**

An object of class `spec_tbl_df` (inherits from `tbl_df`, `tbl`, `data.frame`) with 6426 rows and 3 columns.

**Details**

Variables:

- `level` meters
- `date_time` GMT
- hour time in hours after start of April 2010.

Lat/Long: 41.8071N, -71.4012W Request: [https://tidesandcurrents.noaa.gov/api/datagetter?begin\\_date=20200714&end\\_date=20200714&parameters=level\\_meters&product=water\\_level&station=8454&time\\_zone=GMT](https://tidesandcurrents.noaa.gov/api/datagetter?begin_date=20200714&end_date=20200714&parameters=level_meters&product=water_level&station=8454&time_zone=GMT)  
 Other product documentation: <https://tidesandcurrents.noaa.gov/api/> Google Map: <https://www.google.com/maps/place/Prov+71.4012,+14z/data=!4m5!3m4!1s0x89e444e0437e735d:0x69df7c4d48b3b627!8m2!3d41.8071!4d-71.4012>

---

Robot\_stations

*Waypoints on the path of a fictitious robot*

---

**Description**

This is used in *MOSAIC Calculus* examples about splining.

**Usage**

Robot\_stations

**Format**

A data frame with 16 cases, each of which is an x-y location at the indicated time.

- `x,y` X-Y location in mm
- `t` time in seconds

---

Runners	<i>Running times</i>
---------	----------------------

---

**Description**

A longitudinal record of running times. There are 5977 individual runners included here, each with a unique id. Each runner ran the 10-mile race (See `mosaicData::TenMileRace`) on multiple occasions.

**Usage**

Runners

**Format**

Data frame with 24,334 rows

- `year` the occasion on which the runner ran the Cherry Blossom Ten Miler
- `age` the runners age at the time of that occasion
- `gun` the time (in minutes) from the gun to crossing the finish line.
- `net` time between crossing the start line and crossing the finish line. Sometimes this is missing. It's typically smaller than `gun` because it takes time to reach the starting line.
- `sex` as recorded by the race organizers.
- `previous` How many previous occasions did this person run the race.
- `nruns` The total number of runs recorded for this person (including future runs)
- `start_position` A qualitative indication of how close to the front of the pack the person was positioned at the time the gun went off.

**Author(s)**

Daniel Kaplan

**Source**

Scraped from the Cherry Blossom web site by patient, unremunerated toil of the author.

---

`simpleYacasIntegrate`    *Interface to integration using Ryacas*

---

**Description**

Interface to integration using Ryacas

**Usage**

`simpleYacasIntegrate(tilde, ...)`

**Arguments**

<code>tilde</code>	The tilde expression for the integration
<code>...</code>	Bindings for parameters in <code>tilde</code>

---

`simplify_fun`    *Turn a 1-line function into an inline formula*

---

**Description**

Turn a 1-line function into an inline formula

**Usage**

`simplify_fun(fun)`

`inline_expr(ex, old, new, env)`

`replace_arg_in_expr(ex, old, new)`

**Arguments**

<code>fun</code>	a function, such as produced by <code>makeFun()</code> .
<code>ex</code>	an expression
<code>old</code>	the name to be replaced as produced by <code>as.name()</code>
<code>new</code>	the name to be substituted in for <code>arg_name</code>
<code>env</code>	the environment for the expression

---

slice_plot	<i>Plot a function of a single variable</i>
------------	---

---

### Description

In a slice plot, there is one independent variable. The graph shows the output of the function versus the independent variable. It's called a slice plot to distinguish it from a contour plot, in which the graph has one axis for each independent variable and the output of the function is shown by color and labels.

### Usage

```
slice_plot(
  ...,
  npts = 101,
  color = "black",
  alpha = 1,
  label_text = "",
  label_x = 1,
  label_vjust = "top",
  label_color = color,
  label_alpha = alpha,
  singularities = numeric(0),
  linewidth = 1
)
```

### Arguments

...	Canonical first three argument: [gg], tilde expression, [domain] as well as any parameters to be assigned or re-assigned.
npts	Integer, number of points at which to evaluate the function.
color	Color of curve
alpha	Alpha of curve
label_text	character string label to place near the graph curve. Default: none.
label_x	number between 0 and 1 indicating the horizontal placement of the label_text.
label_vjust	vertical justification of label. One of "left", "middle", "right", "bottom", "center", "top", "inward", or "outward"
label_color	color of label
label_alpha	alpha of label
singularities	numeric vector of x positions at which to break the graph.
line_width	Thickness of stroke

### Value

ggplot2 layers

**Examples**

```
## Not run:
slice_plot(sin(x) ~ x, domain(x = range(-5, 15)))
f <- makeFun(sin(2*pi*t/P) ~ t)
slice_plot(f(t, P=20) ~ t, domain(t = -5:10), label_text = "Period 20", label_x=0.9)
slice_plot(x^2 ~ x) # Error: no domain specified
slice_plot(cos(x) ~ x, domain(x[-10:10])) # domain will be -10 < x < 10
# see domain

## End(Not run)
```

smoother

*Create a smoothing function approximating a cloud of points***Description**

This is defined in the `mosaic` package: See [smoother](#).

SSA\_2007

*US Mortality table from 2007***Description**

Mortality table from the US Social Security Administration issued in 2007.

**Usage**

```
SSA_2007
```

**Format**

A data frame with 240 rows. Each row corresponds to one age year from 0 to 120.

- Age: The age year. 0 corresponds to birth through one year of age.
- Sex: The sex for which the row is relevant.
- Mortality: The fraction of people of that age who died in 2007.
- LifeExpectancy: The calculated "life expectancy" at that age for that sex.

**Details**

Life expectancy is a measure constructed from a simulation. Start with 100,000 people at birth. Use the mortality at each age to follow the living through successive ages. The "life expectancy at birth" (age 0) is the average age at death of those 100,000 people. For the life expectancy in year  $n$ , consider only that fraction of the original 100,000 who survived to year  $n$ . The life expectancy will be the average time until death for those survivors.

**Description**

- `streamlines()` draws raindrop-shaped paths at a randomized grid of points that follow trajectories
- `flow_field()` draws arrows showing the flow at a grid of points
- `trajectory_euler()` compute an Euler solution. ()

**Usage**

```
streamlines(
  ...,
  dom = NULL,
  npts = 8,
  dt = 0.01,
  nsteps = 10,
  color = "black",
  alpha = 1,
  linewidth = 1,
  endpoint = 1
)

flow_field(..., npts = 8, scale = 0.8, color = "black", alpha = 1)

trajectory_euler(..., dt = 0.01, nsteps = 4, full = TRUE, every = 1)
```

**Arguments**

<code>...</code>	The first arguments should describe the dynamics. See details.
<code>npts</code>	The number of points on an edge of the grid
<code>dt</code>	time step (e.g. 0.01)
<code>nsteps</code>	how many Euler steps to take
<code>color</code>	What color to use
<code>alpha</code>	What alpha to use (relative to default)
<code>linewidth</code>	control for the with width of the segments relative to the default.
<code>scale</code>	Number indicating how long to draw arrows. By default, the longest arrows take up a full box in the grid
<code>full</code>	report the derivative and the step size for each variable
<code>every</code>	n, report will contain every nth step

## Details

The dynamical functions themselves will be formulas like  $dx \sim a*x*y$  and  $dy \sim y/x$ . Initial conditions will be arguments of the form  $x=3$  and  $y=4$ . If there are parameters in the dynamical functions, you should also add the parameter values, for instance  $a=2$ .

For `flow_field()` and `streamlines()` you do not need to specify initial conditions. The grid will be set by the domain argument.

The graphics functions are all arranged to accept, if given, a ggplot object piped in. The new graphics layer will be drawn on top of that. If there is no ggplot object piped in, then the graphics will be made as a first layer, which can optionally be piped into other ggformula functions or + into ggplot layers.

## Examples

```
streamlines(dx ~ x+y, dy~ x-y, domain(x=0:6, y=0:3))
flow_field(dx ~ x+y, dy~ x-y, domain(x=0:6, y=0:3))
Dyn <- makeODE(dx ~ 0.06*x, dy ~-x - y, domain(t=0:20), dt=0.1,x=3, y=4)
streamlines(Dyn, domain(x=-5:5, y=-5:5), npts=15)
flow_field(Dyn, domain(x=-6:6, y=-10:10))
trajectory_euler(dx ~ -y, dy ~ .5*x, x=3, y=3)
rabbits <- drabbit ~ 0.2*rabbit - 0.01*rabbit*fox
foxes <- dfox ~ -.2*fox + 0.0005*rabbit*fox
trajectory_euler(rabbits, foxes, rabbit=100, fox=3, dt=0.1, nsteps=500, every=10)
```

---

surface\_plot

*Make an interactive plotly plot of a function of two variables*

---

## Description

An interactive plot lets you interrogate the plot to get numerical values at each point. When type = "surface", the plot can be rotated to see the "shape" of the function from various perspectives. Using the interactive controls, you can save the plot as a PNG file. But it's not possible to overlay plots the way you can with `contourPlot()`.

## Usage

```
surface_plot(
  formula,
  domain = c(-5, 5),
  npts = 50,
  type = c("both", "surface", "contour", "heatmap")
)

interactive_plot(
  formula,
  domain = c(-5, 5),
  npts = 50,
```

```

    type = c("both", "surface", "contour", "heatmap")
  )

  surface_with_contours(formula, domain = c(-5, 5), npts = 50)

```

### Arguments

formula	a formula describing a function in the manner of <code>mosaicCalc::makeFun()</code>
domain	a call to the <code>domain()</code> function giving ranges using the same independent variables as in the formula
npts	The fineness at which to evaluate the function specified by the formula in the plot. Default: 50
type	Plot type: "surface", "contour", "both", or "heatmap"

### Examples

```

## Not run:
interactive_plot(
  sin(fred*ginger) ~ fred + ginger,
  domain(fred=range(0,pi),
        ginger = range(0, pi)),
  type = "both")

## End(Not run)

```

---

 symbolicD

*Symbolic Derivatives*


---

### Description

Constructs symbolic derivatives of some mathematical expressions

### Usage

```
symbolicD(tilde, ..., .order)
```

### Arguments

tilde	a tilde expression with the function call on the left side and the w.r.t. variables on the right side.
...	additional parameters, typically default values for mathematical parameters
.order	a number specifying the order of a derivative with respect to a single variable

**Details**

Uses the Derivs package for constructing the derivative. The `.order` argument is just for convenience when programming high-order derivatives, e.g. the 5th derivative w.r.t. one variable.

When re-assigning default values for arguments in a function being called, as in `D(dnorm(x, mean=3) ~ x)`, you will get a numerical derivative even when the analytic form is known. To avoid this (when possible) use `D(dnorm(x) ~ x, mean=3)`

**Value**

a function implementing the derivative

**Author(s)**

Daniel Kaplan (<kaplan@macalester.edu>)

**See Also**

[D](#), [numD](#), [antiD](#), [plotFun](#)

**Examples**

```
symbolicD( a*x^2 ~ x)
symbolicD( a*x^2 ~ x&x)
symbolicD( a*sin(x)~x, .order=4)
symbolicD( a*x^2*y+b*y ~ x, a=10, b=100 )
symbolicD( dnorm(x, mn, sd) ~ x, mn=3, sd=2)
```

---

| traj\_plot | *Plots a trajectory* |

---

**Description**

This function handles trajectories can stem from either of two sources:

1. A parametric description of a curve, such as  $\sin(t) \sim \cos(t)$ , along with a domain in  $t$ .
2. The solution to an ordinary differential equation as produce by `integrateODE()`

**Usage**

```
traj_plot(..., npts = 500, nt = 5)
```

**Arguments**

...	Handles the first several objects which are, in this order <ul style="list-style-type: none"> <li>• optionally, a ggplot2 object to which this function will add a layer.</li> <li>• tilde: a two sided tilde expression</li> <li>• soln: optionally, a solution object such as from <code>integrateODE()</code>, or instead</li> <li>• domain: a domain object, e.g. <code>domain(t=0:10)</code></li> <li>• Optionally: Graphics parameters to be handed to <code>gf_path()</code>, e.g. <code>color="red", alpha = 0.5, line_width = 2</code></li> </ul>
npts	number of plotted points (default: 500)
nt	number of tick marks to use in a trajectory plot

**Details**

The tilde expression is the critical part

**Examples**

```
traj_plot(2*x + 3 ~ sin(x), domain(x=0:10))
PPdyn <- makeODE(dR ~ 0.3*R - 0.03*R*F, dF ~ -0.3*F + 0.0003*R*F)
Soln <- integrateODE(PPdyn, domain(t=0:20), R=1200, F=8)
traj_plot(R(t) ~ F(t), Soln, nt=10)
traj_plot(R(t)*F(t) ~ t, Soln, nt=0)
```

---

traj_plot_3D	<i>Simple 3D plot of a trajectory</i>
--------------	---------------------------------------

---

**Description**

Takes a trajectory with three state variables as produced by `integrateODE()` and plots out in a 3-dimensional perspective plot, which can be rotated.

**Usage**

```
traj_plot_3D(x, y, z, soln, domain = NULL, npts = 1000)
```

**Arguments**

x	Name of one of the state variables to be plotted.
y	Similar to x
z	Similar to y and x
soln	Solution output from <code>integrateODE()</code>
domain	Optional list like <code>domain=domain(t=c(0,100))</code> . By default, this will be inferred from <code>soln</code>
npts	Number of points at which to evaluate the solution.

**Examples**

```
Lorenz <- makeODE(dx ~ sigma*(y-x), dy ~ (x*(rho-z) - y), dz ~ (x*y - beta*z),
                 rho=28, sigma=10, beta = 8/3)
T1 <- integrateODE(Lorenz, domain(t=0:50), x=-5, y=-7, z=19.4)
traj_plot_3D(x, y, z, T1, npts=5000)
```

---

UK\_GDP

*Gross Domestic Product of the United Kingdom over a millenium*


---

**Description**

An estimate of the real GDP of the United Kingdom over about 7 centuries. "Real" means adjusted for inflation. Take such estimates with a large amount of salt, since pre-industrial era data is very limited and because the components of GDP change substantially over a few decades, let alone a few centuries. Also, the name UK\_GDP is anachronistic, since for most of the record is before the UK was created as a political entity.

**Usage**

```
data(UK_GDP)
```

**Format**

A data frame with 747 rows

- year Which year the estimate is for.
- GDP *per capita* Gross domestic product in GBP (pounds)

---

unbound

*Identifying unbound inputs to a function*


---

**Description**

unbound() finds if there are any unbound parameters in a function. This can be useful for checking before handing a function over to a numerical routine. bind\_params() lets you add parameter bindings or override existing ones.

**Usage**

```
unbound(f)
```

```
bind_params(f, ...)
```

**Arguments**

f                    a function (not a tilde expr.)  
...                   bindings for parameters

---

`US_income`*Income distribution data from the US in 2009*

---

**Description**

Data such as these are used to study income inequality. One measure of this is the "Gini coefficient," which can be estimated from a Lorenz function fitted to such data.

**Usage**`US_income`**Format**

An object of class `tbl_df` (inherits from `tbl`, `data.frame`) with 6 rows and 2 columns.

**Source**

La Haye and Zizler (2021) "The Lorenz Curve in the Classroom", *The American Statistician*, 75(2):217-225]

Variables:

- `income` The fraction of total household income that goes to the corresponding poorest fraction of the population.
- `pop` The fraction of the population whose aggregated income is reported as the `income` value.

---

`vec`*Utilities for vector/matrix calculations*

---

**Description**

Convenience functions for basic operations relating to vector projection. These use the *MOSAIC Calc* conventions that require vectors to be one column matrices.

**Usage**`vec(...)``rvec(n = 3L, fun = rnorm, ...)``M_cols(M, fun = "center")``data_M(.data, tilde)``veclen(v)`

```

v %cang% w

Q(A)

R_for_Q(A)

vbind(..., deparse.level = 1)

vec_names(x, value)

u %dot% b

b %onto% A

b %perp% A

normalize(A)

as_magnitude(A, metric = c("2", "0", "I", "F", "M"))

rmat(nrow = 3, ncol = 6, fun = rnorm)

```

### Arguments

fun	either a function producing random numbers or, for <code>M_cols()</code> the name of action to perform on each column.
A	a matrix
u	a row vector, but a column vector is acceptable too
b	a column vector
metric	metric to use for matrix norm
nrow	number of rows for matrix
ncol	number of columns for matrix
...	values to put in <code>vec()</code> <b>or</b> params for random number generator

### Details

- `vec(1, 2, 3)` makes a **column** vector
- `rvec(n)` random vector with elements
- `vec_names(v, c("one", "two", ...))` gives names to the vectors in a matrix.
- `vbind()` collects column vectors into a matrix. Will also concatenate matrices by column.
- `M_cols()` operates on the columns of a matrix one-by-one. Operations include "center", "unit", "unitcenter", which scale the columns, and "mean", "var", "vlen" which return a scalar for each column.)
- `%dot%` infix operator for dot product. Raw R vectors will work

- %cang% like %dot% but returns the cosine of the angle
- %onto infix projects left arg onto space (vector or matrix) defined by right argument.
- %perp% infix. Like %onto% but returns the reciprocal.
- vbind() collects column vectors into a matrix
- Q(A) the Q part of a qr decomposition
- R\_from\_Q(A) the R part of a qr decomposition. Can apply directly to the output of Q(A)

Convenience functions for basic operations relating to vector projection. These use the *MOSAIC Calc* conventions that require vectors to be one column matrices.

---

vector_arg	<i>convert a function with separate arguments into one with a single vector argument For use with optim.</i>
------------	--

---

### Description

convert a function with separate arguments into one with a single vector argument For use with optim.

### Usage

```
vector_arg(f)
```

### Arguments

f                    a function with multiple arguments

---

Vowel_ee	<i>Recordings of vowel sounds</i>
----------	-----------------------------------

---

### Description

"Oh" refers to "o" as in "stone." "Ee" to "eel".

### Usage

```
Vowel_ee
```

```
Vowel_oh
```

```
Oh_sound
```

```
Ee_sound
```

**Format**

Data frames with two columns

- y: The instantaneous amplitude of the vowel sound.
- t: Time (seconds)

An object of class `tbl_df` (inherits from `tbl`, `data.frame`) with 780 rows and 2 columns.

An object of class `tbl_df` (inherits from `tbl`, `data.frame`) with 7192 rows and 2 columns.

An object of class `tbl_df` (inherits from `tbl`, `data.frame`) with 10194 rows and 2 columns.

**Details**

- `Oh_sound` and `Ee_sound` contain the complete utterances.
- `Vowel_ee` and `Vowel_oh` are snippets of about 0.04 seconds duration.

---

World\_oil

*Yearly production of oil, globally*

---

**Description**

Amount of oil in TWh: terra-watt-hours

Amount of oil in TWh: terra-watt-hours

**Usage**

World\_oil

World\_oil

**Format**

A data frame with 7 rows and numerical variables pressure and volume.

A data frame with 7 rows and numerical variables pressure and volume.

**Source**

[https://ourworldindata.org/grapher/oil-production-by-country?country=OWID\\_WRL](https://ourworldindata.org/grapher/oil-production-by-country?country=OWID_WRL)

[https://ourworldindata.org/grapher/oil-production-by-country?country=OWID\\_WRL](https://ourworldindata.org/grapher/oil-production-by-country?country=OWID_WRL)

---

Zeros

*Finds zeros of a function within a specified domain*

---

**Description**

Finds zeros of a function within a specified domain

**Usage**

```
Zeros(tilde, domain = NULL, ..., nsegs = 131)
```

**Arguments**

tilde	tilde expression defining a function, suitable for makeFun()
domain	specification of domain, as in slice_plot()
...	Assignments to parameters
nsegs	Subdivide the domain into this many segments, looking for a zero in each of those segments. This helps to find multiple zeros.

**Value**

A data frame with two columns. The first has the name of the input in the tilde expression, and gives the values for that input at which the function is approximately zero. The second column, `.output.` gives the actual value of the function at the inputs in the first column.

**Examples**

```
Zeros(a*x + b ~ x, a=1, b=2)
```

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